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1. Introduction

Rare-earth (RE) doped laser materials have attracted enormous interests because of their abundant transition channels and sharp luminescence bands.^{1–5} The potential applications have been widely investigated in a diversity of fields, such as optical imaging, quantum cascade lasers, high-density optical data storage and biophotonic areas.^{6–9} A recent study reveals that the directly pumped Ho³⁺-doped silica microsphere may be an excellent candidate for fabricating 2 µm laser, which can serve as laser-emitting source for mid-infrared telecommunications.¹⁰

Trivalent holmium ion $(4f^{10} \text{ configuration})$ is a greatly promising laser ion due to the substantial transition channels at various wavelengths in the UV, visible and infrared regions.^{11,12} A well-known emission transition ${}^{5}I_{7} \rightarrow {}^{5}I_{8}$ with wavelength near 2 µm of Ho³⁺ can serve as the so called "eye safe" solid-state laser system.¹³ Yttrium oxide (Y₂O₃) is a typical cubic phase crystal structure with $Ia\bar{3}$ space group, which possesses low phonon energy and desirable physical properties including low thermal expansion, high melting point and photochemical stability.¹⁴⁻¹⁶ The Y³⁺ ions of yttrium oxide crystal are six-fold coordinated to nearest O²⁻ ligands, forming

The geometrical structure and electronic properties of trivalent Ho³⁺ doped Y₂O₃ crystals: a first-principles study

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Trivalent rare-earth holmium ion (Ho³⁺) doped yttrium oxide (Y₂O₃) has attracted great research interest owing to its unique optoelectronic properties and excellent performances in many new-type laser devices. But the crystal structures of the Ho³⁺-doped Y₂O₃ system (Y₂O₃ : Ho) are still unclear. Here, we have carried out a first-principle study on the structural evolution of the trivalent Ho³⁺ doped Y₂O₃ by using the CALYPSO structure search method. The results indicate that the lowest-energy structure of Ho³⁺-doped Y₂O₃ possesses a standardized monoclinic *P*2 phase. It is found that the doped Ho³⁺ ion are likely to occupy the sites of Y³⁺ in the host crystal lattice, forming the [HoO₆]⁹⁻ local structure with *C*₂ site symmetry. Electronic structure calculations reveal that the band gap value of Ho³⁺-doped Y₂O₃ is approximately 4.27 eV, suggesting the insulating character of Y₂O₃ : Ho system. These findings could provide fundamental insights to understand the atomic interactions in crystals as well as the information of electronic properties for other rare-earth-doped materials.

> a $[YO_6]^{9-}$ local unit with C_2 site symmetry.¹⁷ After being doped with appropriate rare-earth ions, Y2O3 crystals can serve as excellent laser host materials because of their high thermal conductivity and low phonon energy.¹⁸ In recent years, Ho³⁺doped Y₂O₃ (Y₂O₃ : Ho) crystal has been the subject of intensive investigations as a great promising laser material.¹⁹ Laversenne et al. first demonstrated the growth of Ho³⁺-doped Y₂O₃ single crystal by using the Laser Heated Pedestal Growth (LHPG) technique.²⁰ In addition, they especially analyzed the dynamical laser resonant characteristics of Y2O3 : Ho. Qin et al. studied the luminescence spectra of Ho³⁺-doped Y₂O₃ under the excitation of a 532 nm continuous-wave laser.²¹ The results indicate that Ho³⁺ ion possesses several fluorescence transitions in the ultraviolet and violet region (306, 390 and 428 nm) which are assigned to the transitions of ${}^3D_3 \rightarrow {}^5I_8, {}^5G_4 \rightarrow {}^5I_8$ and ${}^5G_5 \rightarrow$ ⁵I₈, respectively. Wang *et al.* reported a high output laser operation at around 2.1 μ m of Y₂O₃ : Ho with low scattering loss and excellent optical quality.²² Their results revealed that Ho³⁺doped Y₂O₃ system shows attractive prospect in high-power and efficient laser applications as a laser gain medium. Although numerous investigations have been widely reported on Y_2O_3 : Ho, there is no systematic study to elucidate its microstructure and electronic properties.

> In this paper, we perform extensive structure searches to obtain the ground-state structure of Y_2O_3 : Ho based on the CALYPSO (Crystal structure AnaLYsis by Particle Swarm Optimization)²³⁻²⁷ method coupled with the DFT (density functional theory). Furthermore, we calculate and analyze the band structure, density of states and the ELF (electron localized

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function) to gain deeper insights into the electronic properties of Ho^{3+} -doped Y_2O_3 system. The outline of this paper is organized as follows. We exhibit a brief description of the calculation method in Section 2. In Section 3, we present our results and discussion. A conclusion is finally given in Section 4.

2. Computational methods

We have explored the structural evolution of Y_2O_3 : Ho crystal by using the unbiased CALYPSO²³⁻²⁷ method. The CALYPSO is a reliable structure prediction method which has been validated by a large variety of crystal structures.²⁸⁻³² We perform an evolutionary variable-cell structure prediction with 80 atoms per simulation cell at ambient pressure. To determine the most stable structure of Y_2O_3 : Ho system, we optimized the all lowest-lying candidate structures by using the density functional theory in VASP (Vienna Ab Initio Simulation Package) code.33-35 The frozen-core all electron projector-augmented wave (PAW) method has been adopted, with 4f¹¹5s²5p⁶6s², 4d¹5s² and 2s²2p⁴ treated as valence electrons for Ho, Y and O, respectively. For describing the influence of the correlation effect introduced by 4f electrons of Ho atoms, we employ the local density approximation (LDA) with an onsite Coulomb repulsion parameter U36 to determine the electronic band structure of Y₂O₃ : Ho. The U value of Ho has been determined to be 6.8 eV by Min et al.37 Phonon dispersion curve have been calculated by the PHONOPY code.38

3. Results and discussion

We carefully examine the ground-state crystal structure of Ho^{3^+} doped Y_2O_3 by using the unbiased CALYPSO structure search method with the stoichiometric ratio of Ho : Y : O = 1 : 31 : 48 under ambient conditions. The lowest-energy structure of Y_2O_3 : Ho is successfully identified and displayed in Fig. 1. It can be seen from Fig. 1 that the ground-state Y_2O_3 : Ho crystal possesses the monoclinic configuration with the Ho³⁺ ion (0.901 Å) substitute for Y³⁺ ion (0.900 Å) in the Y₂O₃ host. The concentration of the impurity Ho³⁺ is equal to 3.125%, which is in excellent agreement with the result measured by Atabaev



Fig. 1 Crystal structure and $[HoO_6]^{9-}$ local unit of the ground-state Ho^{3+} -doped Y_2O_3 . The bond lengths are in the unit of Å.

*et al.*¹⁵ The site symmetry of $[HOO_6]^{9-}$ local structure are calculated to be C₂ and Ho³⁺ position are six-coordinated by oxygen atoms. The three different bond lengths between Ho–O bonds are calculated to be 2.214, 2.232, and 2.318 Å, respectively. The Ho³⁺ doped Y₂O₃ crystal structure belongs to the standardized *P*2 symmetry and the calculated lattice constants are a = b = c = 10.524 Å, $\beta = 90^{\circ}$. The coordinates of all atoms for the ground state Y₂O₃ : Ho are summarized in Table 1 for further investigations. Moreover, our structure searches also predict many metastable structures of Ho³⁺-doped Y₂O₃ which can play important roles to explore the structural evolutions. The first four optimized low-lying structures (a), (b), (c) and (d)

Table 1 Coordinates of all atoms for the ground state ${\rm Ho}^{3+}{\rm -doped}$ Y_2O_3

Atom	x	у	z	Wyckoff site symmetry
Но	0.50000	0.03244	-0.00000	1c
O(1)	-0.09805	0.37926	0.39177	2e
O(2)	0.40264	0.87998	0.89255	2e
O(5)	0.40205	0.12070	0.60831	2e
O(6)	-0.09818	0.62076	0.10808	2e
O(9)	0.14194	0.15190	0.37921	2e
O(10)	0.64186	0.65190	0.87934	2e
O(11)	-0.14176	0.84802	0.87927	2e
O(12)	0.35819	0.34791	0.37925	2e
O(17)	0.12928	0.39175	0.15192	2e
O(18)	0.62931	0.89190	0.65180	2e
O(19)	0.62868	0.10740	0.84899	2e
O(20)	0.12931	0.60822	0.34801	2e
O(25)	0.59794	0.62079	0.60830	2e
O(26)	0.09816	0.12070	0.10813	2e
O(29)	0.09808	0.87932	0.39175	2e
O(30)	0.59805	0.37924	0.89184	2e
O(33)	0.35820	0.84806	0.62099	2e
O(34)	-0.14195	0.34792	0.12077	2e
O(35)	0.64100	0.15153	0.12014	2e
O(36)	0.14182	0.65199	0.62075	2e
O(41)	0.37080	0.60848	0.84803	2e
O(42)	-0.12934	0.10816	0.34796	2e
O(43)	-0.12943	0.89189	0.15191	2e
O(44)	0.37080	0.39172	0.65211	2e
Y(1)	0.00004	0.24998	0.24998	2e
Y(2)	0.49982	0.75036	0.24967	2e
Y(3)	0.50011	0.24976	0.75024	2e
Y(4)	0.00005	0.75003	0.75003	2e
Y(9)	-0.25020	0.24979	0.96795	2e
Y(10)	-0.25006	0.75002	0.53211	2e
Y(13)	0.71753	0.00010	0.24967	2e
Y(15)	-0.21791	0.50004	0.24995	2e
Y(21)	-0.25026	0.75030	0.03217	2e
Y(22)	0.25002	0.24997	0.53226	2e
Y(25)	-0.21803	0.00001	0.75019	2e
Y(26)	0.28218	0.49999	0.24999	2e
Y(17)	0.00000	0.96781	0.00000	1a
Y(30)	0.00000	0.46784	0.00000	1a
Y(18)	0.50000	0.53219	-0.00000	1c
Y(19)	0.00000	0.03214	0.50000	1b
Y(29)	0.00000	0.53220	0.50000	1b
Y(20)	0.50000	0.46778	0.50000	1d
Y(31)	0.50000	0.96786	0.50000	1d



Fig. 2 Coordination structures of the metastable (a–d) for Y_2O_3 : Ho.



Fig. 3 Comparison of the simulated XRD spectrum for the ground-state and metastable (a–d) Y_2O_3 : Ho with experimental patterns.

from low to high energy are exhibited in Fig. 2. It is found that the Y^{3+} ions of these isomers are replaced by Ho³⁺ ions at different sites in the host crystals. Interestingly, the isomer (a) possesses the same *P*2 monoclinic configuration with the ground-state structure while the isomers (b), (c) and (d) exhibit the *P*1 space group. In these metastable structures, we find that the impurity Ho³⁺ ions tend to occupy the crystal face site positions of the Y^{3+} .

To clarify the true structure of the ground-state Y₂O₃ : Ho, as shown in Fig. 3, we calculate the X-ray diffraction (XRD) patterns of the ground state structure. We can clearly see from Fig. 3 that the simulated spectrum of Y₂O₃: Ho are in good accordance with the observations in experiment.¹⁵ In addition, the XRD patterns of the four metastable structures are calculated and the results are also plotted in Fig. 3. It can be seen from Fig. 3 that the overall distribution of the peaks is similar, suggesting that the structural parameters of the four metastable structures are close to each other. To further validate the dynamical stability of Ho³⁺-doped Y₂O₃ system, we have calculated the phonon dispersion curves in Fig. 4 and no imaginary phonon frequencies can be seen over the entire Brillouin zones. The result indicates that the determined ground-state structure of Ho^{3+} -doped Y_2O_3 crystal is dynamically stable. These theoretical results provide great support for the reliability of our structural prediction methodology.

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Fig. 4 Calculated phonon dispersion curve for the ground-state Y_2O_3 : Ho.

We have calculated the electronic band structure and the total as well as partial DOS for Y_2O_3 : Ho. As illustrated in Fig. 5(a), the direct band gap value for Ho³⁺-doped Y_2O_3 is about 4.27 eV at the Γ point, which is approximately 2/3 of the experimental value ($E_g = 6.2$ eV) determined by Wallace and



Fig. 5 The calculated (a) electronic band structure and (b) total as well as partial densities of states of Y_2O_3 : Ho.



Fig. 6 ELF maps of (a) the structure and (b) $\langle 100 \rangle$ plane for the ground-state Y_2O_3 : Ho.

Wilk.39 This result can be ascribed to the general underestimation of band gap value by the first-principle calculations. The result indicates that the Ho³⁺ impurity ion remains the insulating character of Ho^{3+} -doped Y₂O₃ crystal. From Fig. 5(b), we can clearly see that the low valence band region is mainly composed of p states with the smaller contributors of d states ranging from -1 eV to 0 eV, and the dominant contributions of the high conduction band between 4.3 eV to 9 eV are mainly occupied by p, d and f states. It should be noted that the s states is very weak from -1 eV to 9 eV. In addition, we have calculated the electron localized function (ELF) to visualize the chemical bonding character in Y2O3: Ho crystal. The ELF in crystal structure and the ELF of the (100) plane are presented in Fig. 6. It is shown that the ELF near the Y and Ho atoms value is close to 0.9, which suggests that the electrons are extremely localized around the Y and Ho atoms.

4. Conclusions

In summary, we have explored the ground-state crystal structure of Ho^{3+} -doped Y_2O_3 by means of the unbiased CALYPSO method combined with first-principle calculations. It is shown that the ground-state Y_2O_3 : Ho structure possesses a novel *P*2

phase with the monoclinic symmetry. We carry out a systematical investigation to the microstructure evolutions for the ground-state Y_2O_3 : Ho crystal. The results indicate that the impurity Ho³⁺ ion substitutes the positions of Y³⁺ ions in the host crystal lattice, forming the $[HOO_6]^{9-}$ local structure. We find that the impurity Ho³⁺ ions tend to occupy the crystal face positions of the Y³⁺ ions from the structural features of the ground-state and metastable structures. We further calculate the band structure and density of states by LDA + *U* method for Y_2O_3 : Ho. Our result reveals that the electronic band gap of Ho³⁺-doped Y_2O_3 is 4.27 eV. We hope that these findings can provide valuable guidance for future experiment research of Y_2O_3 : Ho.

Conflicts of interest

The authors declare no competing interests.

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