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# Elastic theory for the deformation of a spherical dielectric biological object under electro-optical trapping

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**Abstract:** The shear modulus of a dielectric spherical particle is investigated using a combination of triangular (or square) electrodes and a single-beam optical tweezer. The electronic response of a spherical dielectric particle is dominated by the local interactions with the trapping beams. Positive dielectrophoresis on a dielectric particle works at high frequencies. By measuring the geometrical parameters of the sphere as a function of the applied voltage, the elasticity of sphere is determined theoretically from the maximum applied voltage when a particle escapes to the electrode from trapping centre. To check the validity and efficiency of the elasticity derived mathematically, similar experimental results got by other techniques have been studied. The method is suggested to be a general tool to distinguish healthy and diseased cells.

**Key words:** Dieelctrophoresis, Dielectric particle, Optical tweezers, Shear modulus

## 1. Introduction

Dielectrophoresis (DEP) is an electrokinetic movement of neutral particles induced by polarization in non-uniform electric field *(1).* The magnitude and direction of the induced dielectrophoretic force acting on a particle in a non-uniform electric field are dependent on the characteristics of the applied electric field as well as the dielectric properties of the surrounding medium and of the particle itself. The strength of DEP depends strongly on the medium, the electrical properties of the particle (permittivity and conductivity), the particle's shape and size, and the frequency of the electric field. Varying of any parameter allows the manipulation of particles with great selectivity, which can be utilized in many applications. In the usual case, when the permittivity of the particle is higher than that of the medium, the electric dipole moment vector of the particle is aligned parallel with the electric field, and the particle is attracted towards higher field strengths.

In biological applications, when the particles are living cells, the field generated should be large enough to polarise the cells with a significant induced dipole moment, but not so large as to cause their electrical or thermal damage. On the other hand, the electrode geometry should ensure a highly non-uniform field to give rise a DEP force that overcomes the randomising effects of Brownian motion in the surrounding medium, and to cause the cell to move to a desired location.

If the external field varies linearly over the characteristic particle dimensions, the polarization of a dielectric sphere in an electric field can be represented by replacing the particle with an equivalent *effective* dipole located at the centre of the particle. The DEP force on a dipole is proportional to its dipole moment and the gradient of the electric field. The DEP response exhibited by a typical mammalian cell depends also on the frequency of the applied electric field. In early studies, it was observed that at low frequencies (less than 1 kHz) the DEP behaviour is influenced by the cell surface charge, implying that the *effective* polarizability of the cell is dominated by the electrical double layer that surrounds a charged cell *(2,3).*

Theoretical studies have predicted that a change in shape from a sphere to an ellipsoid can result in significant changes of the DEP force *(4).* The main reason is that the membrane of cells acts as a capacitor, because it is consisted of a thin dielectric situated between two ionic conductors (e. g. the outer (PBS) and inner electrolytes (cytoplasm). The bigger the cell, the longer it will take to polarise the cell membrane using the fixed ion charge density available in the surrounding electrolyte. A longer period in the time-domain translates to a lower frequency in the frequency-domain.

An important practical application of the influence of cell size and membrane capacitance on the dielectric polarizability of a cell has recently been demonstrated by Holmes et al. *(5)*. The applied electric field polarizes the dielectric particle, and the poles, then experience a force along the field lines, which can be either attractive or repulsive according to the orientation of the dipole. Since the field is non-uniform, the pole experiencing the greatest electric field will dominate over the other, and the particle will move. According to Maxwell-Sillars polarization, the polarization of a dielectric particle in an inhomogeneous medium is frequency dependent, and its dipole orientation is determined by the relative polarizability. The DEP force direction depends on field gradient rather than field direction, it occurs in AC as well as DC electric fields, polarization and the direction of the force will depend on the relative polarizabilites of the dielectric particle and medium. When a dielectric object moves towards higher field strengths is referred to as positive DEP (pDEP), and if it moves away from the high field regions, it is known as negative DEP (nDEP).

When a prolate, oblate or triaxial ellipsoidal particle is polarized, the dipole moment will align the particle with its dipole parallel to the field lines (6-11). Its orientation is a function of the electric field frequency and the dielectric properties of the medium and particle. DEP can also be used to rotate a particle, or trap it as well. Similarly to DEP, optical tweezers can also be used to trap particles via balancing of gradient and scattering forces. By optical tweezers, it is possible to trap particle close to the electrode or anywhere between the electrodes.

## 2. Theoretical methods

The aim of this study is to find the deformation and stress of a sphere, which is dielectric in nature, subjected to a known set of surface tractions. There are two steps for this study; first is to design the electrode where a dielectric object will trap by a laser tweezers, and second, based on escaping velocity of a particle, an equation of elastic modulus of a prolate/oblate spheroid has been derived.

### *2.1. Electrode design*

Electrodes (Fig. I) can be fabricated on a glass substrate by vacuum deposition of conducting material. The thickness of the metal laminate on the glass surface is $d_{el}$ µm (~50-100 µm). The shape of the electrodes may be either triangular or square, and the space between them can be filled with electrolyte. The gap between two electrodes is $d_{el}$ µm and the length of each side of the triangular electrodes is $d_{el}$ µm. Electrodes can be connected to a function generator. Laser light with an optical axis perpendicular to the substrate focuses in between the electrodes, forming an optical tweezers to trap a particle.

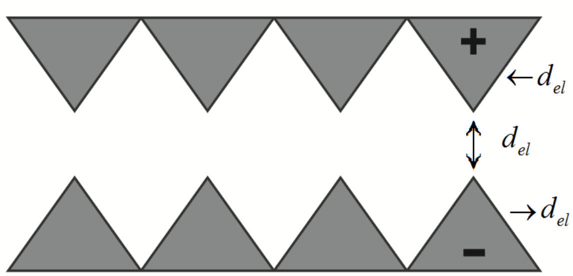


Figure I. Triangular electrode

*2.2. Theoretical background of the shear modulus*

Dielectrophoretic force (DEP) on a spherical dielectric particle in a non-uniform electric field will be time average along with electric field intensity gradient in three dimensions. When a spheroidal object is suspended in a hypotonic solution on conductive electrodes, the time - averaged DEP force is

$$\langle F_{DEP} \rangle = \frac{3}{4} \upsilon \varepsilon_m f_{CM} \nabla |E|^2$$

Where $\upsilon$ is the volume of a sphere, of which radius is $R_o$ immersed in a phosphate buffer solution of permittivity $\varepsilon_m$, which is very low conducting and subject to the gradient of the magnitude of non-uniform electric field squared, $E^2$. $f_{CM}$ is the frequency dependent Clausius-Mossotti factor, which depends on the shape of the particle. In general, $f_{CM}$ is bounded by 1 and -0.5. At high frequencies, $f_{CM}$ =1, when the particle is more polarized than medium *(12).*

When a particle is trapped by a single beam optical tweezer (Fig. II), the gradient force resembles the dielectrophoretic force and the derivative of the potential energy with respect to the major axis of prolate/oblate is the energy gradient. The gradient force used by optical tweezers arises from fluctuating electric dipoles that are induced when light passes through birefringent particles, that polarize horizontally and mutually perpendicular with the transverse wave of light, which consequently experience a time-averaged force in the direction of the field gradient. In the case of dielectric objects, for example red blood cells, which are transparent to light, the gradient force predominates over the scattering force. The scattering force levitates and accelerates the particle in solution and the gradient force acts to restore the position of a displaced bead towards the centre of the laser focus. The gradient force approximately satisfies Hooke's law for significantly small displacements with spring constant. The spring constant characterizes how stiff the optical trap is, and it has an alternative name, trap stiffness. Since the two gradient forces compensate each other while the particle is trapped, and the particle is normally released when the beam waist of the tweezers gets out of the cell.

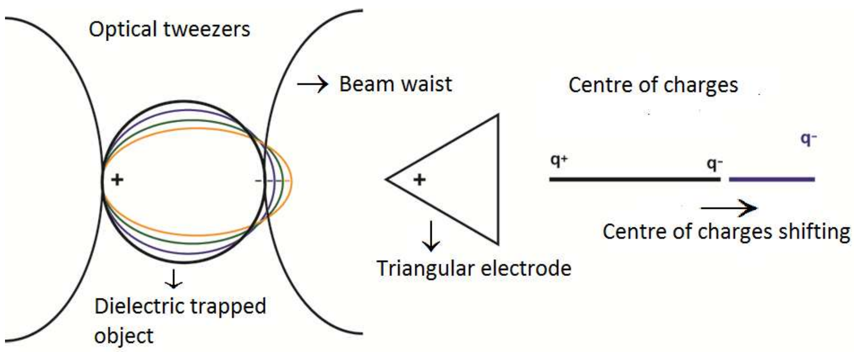


Fig. II. Particle trapped and deformed by external electric field

After the deformation or stretch by an external electric field (Fig.II), the spherical particle can be either prolate or oblate. Let us consider a prolate where $a>b=c$ or an oblate, where $a<b=c$, where $a$, $b$ and $c$ are the main radii of the ellipsoid. For a prolate *(b=c)* aligned with its long axis (*z*) parallel to and positioned on the axis of an axial symmetric field. The pDEP force vector is directed along the gradient of the electric field intensity, which is parallel to the

longest axis of the prolate/oblate spheroid. The simplified and modified form of pDEP force is

$$\left\langle F_{pDEP} \right\rangle = \pi a_{mj} b_{mr}^2 \varepsilon_m \frac{\partial E_{0,z}^2}{\partial z} z □$$

The gradient of the magnitude of the non uniform electric field intensity whose value depends on the shape (e. g. Square, triangular etc.) of electrode because of tip position and its surrounding space, and it is the highest at the edge and the surface of the electrode (pDEP) and exponentially decays, $\exp\left(-\frac{\pi y}{d_{el}}\right)$ away from it, where $y = d_{el}/\pi$ is a constant. Using the scaling law *(13)*, the pDEP can be defined as the following equation

$$\left\langle F_{pDEP} \right\rangle = 0.368 \pi a_{mj} b_{mr}^2 \varepsilon_m \frac{V^2}{d_{el}^3}$$

Where $d_{el}$ is the gap and the width of the electrode and $V$ is the applied voltage with $y = d_{el}/\pi$ the vertical direction.

When a $F_{DEP}$ is applied to an elastic body (e. g. RBCs), the body deforms. The way in which the body deforms depends upon the type of applied forces to it. Shear stress acts tangential to the surface of the material and parallel to the longest axis of prolate/oblate. A shearing stress for a prolate alters only the shape of the elastic body leaving the volume unchanged. In the case of constant volume, and if the sphere is stretched rather than compressed, it tends to contract in the directions transverse to the direction of stretching. For a perfectly incompressible sphere deformed elastically at small strains, the Poisson's ratio is $\frac{\Delta b_{mr}}{\Delta a_{mj}} = 0.5$. For example, RBCs is a quasi-incompressible elastic body whose Poisson's number is 0.5. By applying the Poisson's number for an elastic particle, the eccentricity for its will be 0.866 (Fig. III), which indicates the shape of a prolate spheroid. In the dipole approximation, $F_{pDEP}$ is the net force acting through the centre of charges shifting (Fig. II) of an elementary dipole representing the prolate/oblate which is subject to inhomogeneous electric field.

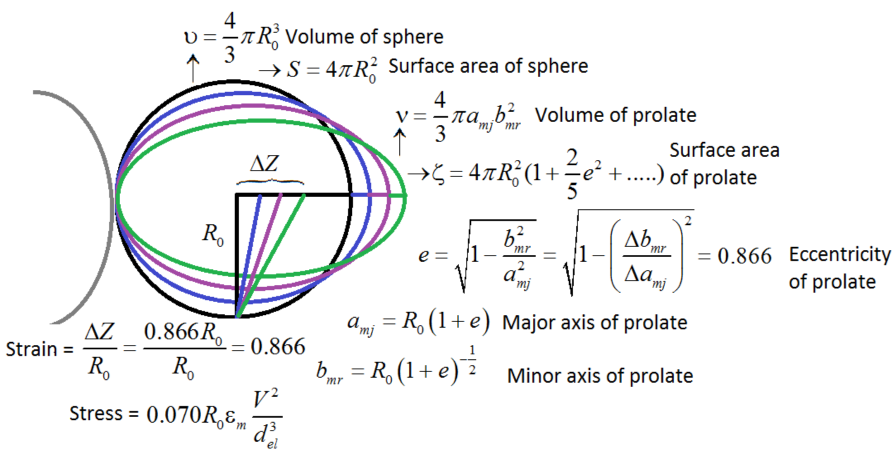


Fig. III. Stress and strain of prolate

Modulus of rigidity ($\mu$), which characterizes the stiffness of the prolate/oblate shape particle, is defined as the ratio of shear stress to the shear strain.

$$\mu = 0.080 R_0 \varepsilon_m \frac{V^2}{d_{el}^3}$$

As the stretching forces by the tweezers and DEP keep balance via the trapped particle, the above value should also be characteristic to the stiffness of the cell, as well. Therefore, the elastic modulus depends on the radius of the spherical cell ($R_0$), the medium ($\varepsilon_m$) where the particle suspends between the gap of the two electrodes ($d_{el}$), and the squared of the applied electric field ($V$). If the membrane thickness of a spherical cell increase, the elasticity of the membrane will increase by increasing $R_0$ and a number 0.080 represents the surface deformation value of a sphere.

**3. Summery and conclusions**

Biological particles, such as cells and some types of viruses, have a complicated internal structure. The common approach to determine the elasticity ($\mu$) theoretically of dielectric spherical biological cells is to use a membrane thickness. A single shelled biological particle is red blood cells (RBCs), which are membrane capsules filled with a concentrated solution of proteins and it is a disk-like shape solid body, can also be trapped by optical tweezers *(14-15)*. The internal stress of a spherical dielectric biological cell (e.g.RBCs) is determined by Hooke's law because of contiguous spectrins network and its surface force is exerted using the optical tool, a single beam optical tweezer.

The knowledge of deformation and stress of cell is of great importance by various physiological and pathological parameters, such as temperature, morphology, osmotic pressure, ATP effect, parasite invasion, genetic diseases (e.g., sickle cell anaemia) (16). Erythrocyte deformability also becomes altered in other diseases such as diabetes mellitus (17-*21*), essential hypertension (*22-26)*, arteriosclerosis, coronary artery disease (*27, 28*) and cancer (*29*). The shear modulus in this study can be employed to find the shape of a spherical particle submerged in a horizontal flow of a liquid. Furthermore, it may be used to determine the maximum stress (and hence the potential fracture) on the surface of a sphere.

Since most cells and molecules show the pDEP motion in low-conductivity media *(30-33)*. The shape of the most cells is roughly cuboidal or spherical. Most human cell size is about to 10-15 μm and cell growth increases volume faster than surface area. Coccus is a sphere shape bacteria. The medium of cell suspending solution is hypotonic, isotonic and hypertonic, which affect the fluid volume of the cell. For example, if RBCs are suspended in hypotonic solution, its volume is changed from biconcave to spherical.

There are numerous data in the literature that the stiffness of healthy cells differ significantly from that of diseased cells. Therefore, this work shows that the theoretical technique is an easy to use and simple alternative method to distinguish healthy and diseased cells. This study can also be utilized to deduce the viscoelastic properties of spherical biological objects, including stem or cancer cells, captured in an optical trap, where deformation can be observed experimentally.

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